MEASURING THE HEIGHT OF TREES WITH THE ABNEY LEVEL

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The purpose of this article is to describe accurate methods by the use of hand instruments in measuring the heights of large trees. Such measurements can be made to an accuracy of 1 inch.

Textbooks on forest mensuration deal too lightly with the problem of the "lean" of a tree and fail to outline precise methods in measuring irregular trees. Tree monarchs are generally more than a century old. In battling storms they have lost limbs and often their leader and have grown up irregularly. In the case of deciduous trees the top is seldom in line with the trunk. If a measurement is taken to the trunk instead of the point beneath the top of the tree, the resulting error is so great that any measurements by instruments become a waste of effort.

The problem of measuring the height of a tree is actually the determination of the length of the perpendicular dropped from the top of the tree to a horizontal plane passing thru the base of the trunk. The first problem is to locate the point on the ground beneath the treetop. Place a tripod in the approximate (tentative) position beneath the treetop. It is necessary to use a tripod or standard of some kind instead of a stake because the distances are so great and the brush is generally so thick that a stake can not be seen. Stand in a position where the treetop can be seen as far as possible from the tree. Hold a vertical level in a position where the line of sight from the treetop touches the side of the level at its top. Follow the vertical line to the ground with your eye and move the tripod to a position where the vertical line passes thru the head of the tripod. Then move to a position about 90° distant, make the same observation of the treetop, and move the tripod so that its head is in line with this vertical line, keeping it in the same vertical plane as in the original location. This final position of the tripod is the point beneath the treetop and is actually the point near the ground on the line of intersection of the two vertical planes. Drop a plumb bob from the head of the tripod to the ground.

After thus locating the point beneath the treetop, the next problem is to measure the length of this vertical line. Various methods are presented, depending on the configuration of the foliage and the topography.

IF THE TREETOP CAN BE SEEN AT AN ANGLE OF 45° AND THE GROUND IS LEVEL OR DOES NOT SLOPE MUCH

Clamp the Abney level at a tangent of 1.00 (100 on the scale). Stand in a position where the treetop and the bubble show up together. Place a stake between your legs. Measure the horizontal distance from this stake to the point beneath the treetop. This is one leg of an isosceles right triangle and is the equivalent of the vertical line extending from the treetop to the horizontal level of your eye.

The next problem is to determine the vertical distance of your eye above the base of the trunk. Place a 6 foot ruler against the trunk in as nearly a vertical position as possible. Hold it in position at the top by sticking an awl into the tree 2 inches below the top of the ruler on one side and another awl 4 inches below the top on the other side. These awls hold the ruler in place and also serve as visual guides in reading distances on the ruler. Clamp the Abney level at 0 elevation (horizontal), sight thru it and read the number of inches above the ground on the ruler. This is the height of your eye above the base of the tree.

In looking at the treetop your head was tilted back and your eyes were 2 inches higher than when looking horizontally.

The height of the tree is the height of your eye above the base of the tree plus 2 inches plus the height of the treetop above the level of your eye.

IF THE TREETOP CANNOT BE SEEN AT AN ELEVATION OF 45° AND THE GROUND IS LEVEL OR DOES NOT SLOPE MUCH

If intervening foliage cuts off the sight of the treetop so that it cannot be seen at an angle of 45° in any position, the problem is still relatively simple.

Stand in a position as close to the tree as the top can be seen and read the tangent on the Abney level. Place a stake between your legs. Measure the horizontal distance from this stake to the point beneath the treetop.

$$\tan = \frac{\text{height}}{\text{horizontal distance}}$$

height = tan x horizontal distance. This is the height of the treetop above the level of your eye.

The height of your eye above the base of the tree is ascertained as in the preceding case. The total height is the height of your eye above the base of the tree plus 2 inches plus the height of the treetop above the level of your eye.

IF THE GROUND SLOPES UPWARD MORE THAN 6 FEET FROM THE BASE OF THE TREE

If the slope is so great that the horizontal line from your eye passes above the top of a 6 foot ruler held against the tree, an accurate measurement by tape of the horizontal distance to the point beneath the treetop usually cannot be taken, and if this is the case, it is necessary to solve two or three triangles.

The problem is illustrated in Fig. 1. Stand in any position G where you can see the treetop. In this case use the degree scale on the Abney level. Point the Abney level at the treetop A and read the angle F. Point the Abney level at the point on the ground B touched by the plumb bob hanging from the tripod and read the angle E. Place a stake between your legs and measure the rectilinear distance d along the ground from this stake to the plumb bob B. It is necessary to know the height of your eye c above the ground when looking downward; if you know the height of your eye when looking horizontally, subtract 1 inch therefrom.

(1) $D = 90° - E$

In the oblique triangle BDG, by the law of the sines:

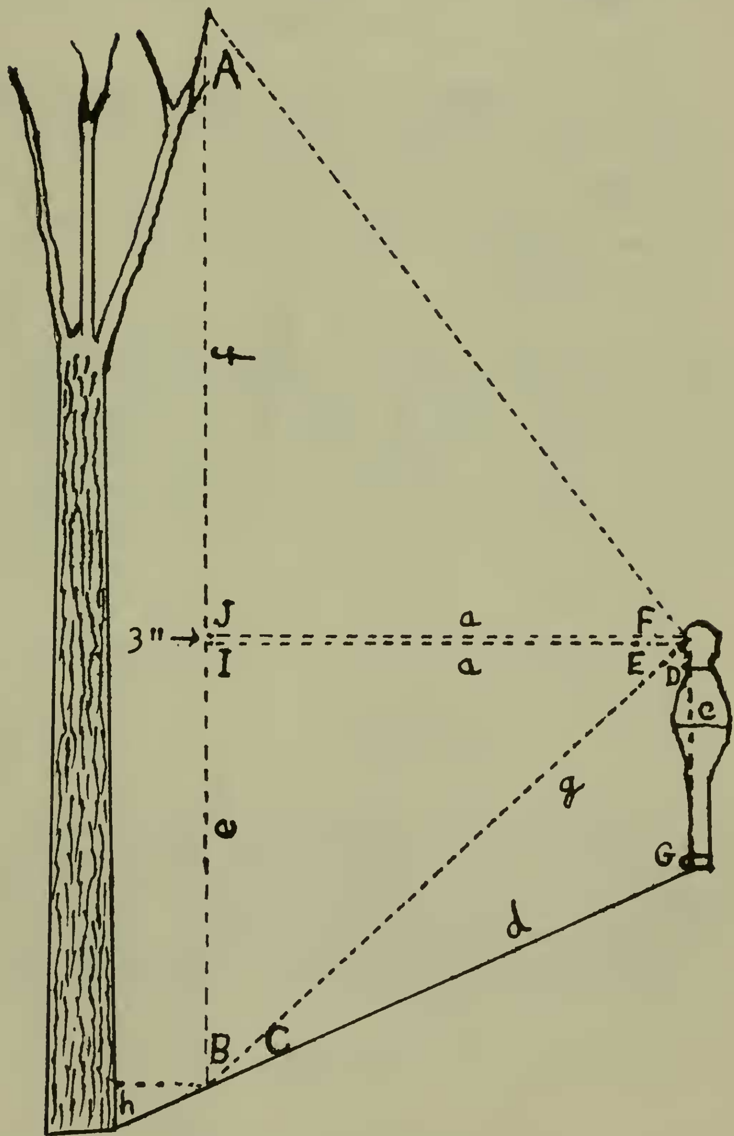

Fig. 1

$$\frac{c}{d} = \frac{\sin C}{\sin D}$$

$$c \sin D = d \sin C$$

$$(2)\ \sin C = \frac{c \sin D}{d}$$

$$(3)\ G = 180^\circ - (C + D)$$

$$\frac{g}{c} = \frac{\sin G}{\sin C}$$

$$g \sin C = c \sin G$$

$$(4)\ g = \frac{c \sin G}{\sin C}$$

We are seeking e and f in the right triangles BEI and AFJ, each of which has a leg a. These triangles are separated by a distance of 3 inches, representing the difference in the elevation of your eye when looking upward and when looking downward. These two triangles can be solved separately for e and f. Knowing E and g in the triangle BEI, we can solve for e and a. If angle F is 45°, $f = a$. But if angle F is not 45°, instead of solving the two triangles separately, the procedure can be shortened by moving the two triangles together geometrically to make the oblique triangle ABD and solving for $e+f$ as follows:

$$(5)\ A = 90^\circ - F$$

$$\frac{g}{e + f} = \frac{\sin A}{\sin (E + F)}$$

$$(e + f) \sin A = g \sin (E + F)$$

$$(6)\ e + f = \frac{g \sin (E + F)}{\sin A}$$

The point B where the plumb bob touches the ground will probably not be on the same horizontal level as the base of the tree and it will be necessary to make an adjustment for the height h above (or possibly below) the level of the base of the tree. This can be done by extending a ruler or steel tape, held in a horizontal position by means of a level, from B to the tree trunk and measuring the vertical distance h on the tree trunk from this point to the base of the tree.

The height of the tree is h plus e plus f plus 3 inches.